B.Sc Part I (Honrs)

Factorisation of mappings.

Definitions :-

Function (or Mapping) :- A function from a non-empty set A to non-empty set B is a rule which associates to each element of A a unique element of B and is denoted by

$f: A \to B$.

The set A is called the domain of the function (or mapping) and the set B is called the Co-domain of the function (or mapping)

Range :- Let $f: A \to B$. The range of f is the set of all those elements of B which are images of at least one element of A. The range of f is also called the image of f and it is denoted by $f(A)$.

This $f(A) = \{y \in B / y = f(x)$ for at least one $x \in A\}$. Evidently $f(A) \subseteq B$.

Discuss a bout Factorisation of a mapping or Canonical decomposition of mapping.

Ans :- Let $f: A \to B$ be a map of a set A into a set B.

Then $R = \{(a_1, a_2) \in A \times A : f(a_1) = f(a_2)\}$ is an equivalence relation in A.

We Consider the following maps.

(i) The map $h: A \to B/R$ given by $h(a) = a$, where $a \in A$ and $[a]$ denotes the equivalence class to which 'a' belongs

Here $h: A \to B/R$ is well defined onto map.

(ii) $g: A/R \to f(A)$ given by $g(a) = f(\dot{a})$

Where $\{a\} = A/R$ and $a \in \{a\}$

i.e. $g: A/R \to f(a)$ is well defined.

If $a_1 \in [a]$ then $a_1 R a$ i.e. $(a_1, a) \in R$

$\Rightarrow f(a_1) = f(a)$. Hence the values of g at an element $\{a\}$ of A/R does not depend on particular representation.

Also we see that g is an one-one map

in fact $f(a) = f(a_1) \Rightarrow (a_1, a) \in R$

$\Rightarrow a_1 R a$ and thus $[a] = [a_1]$

$\therefore [a] \neq [a_1] \Rightarrow g(a) \neq g(a_1)$

i.e. g is a one one and onto map.

(iii) The map $i: f(A) \to B$ given by

$i(f(a)) = f(a) \; \forall \; f(a) \subseteq f(A)$

$\therefore$ i is a one-one map.

Further we have

$\therefore$ $(igh)(a) = (ig)(h(a))$

$= (ig)[(a)] = i(g[a])$

$= i(f(a) = f(a)$

$\therefore$ $f = igh \Rightarrow f = iogoh$

Thus the decomposition of a map into and onto, one one onto and one one map is called a factorisation of f.

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